MULTIMEDIA UNIVERSITY

STUDENT ID NO

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 3, 2021/2022**

## PCM0235 - CALCULUS

(Foundation in Information Technology)

9 AUGUST 2022
9.00 a.m. – 11.00 a.m.
(2 Hours)

---

**INSTRUCTIONS TO STUDENT**

1. This Question paper consists of 2 pages excluding the cover page and appendix.

2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.

3. Write all your answers in the Answer Booklet provided.

**Question 1 [10 marks]**

a. Find the following limits.

i. $\lim_{x\to\infty}\dfrac{73+x^3-x^4}{-0.5+3x^2+7x^3}$ (2 marks)

ii. $\lim_{x\to 0}\left(\dfrac{2x^2-3x+4}{x}+\dfrac{5x-4}{x}\right)$ (3 marks)

b. If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$ and if $f(x)$ is continuous at $x=2$, find the value $k$. (5 marks)

**Question 2 [10 marks]**

a. Find the derivative of the following functions.

i. $y=x^4\left(\sin x^3-\cos x^2\right)$ (2 marks)

ii. $4x^2y^7-2x=x^5+4y^3$ (3 marks)

b. Use integration by parts to evaluate the following integral:

$\int\frac{1}{x^3}\ln x^2\,dx$ (5 marks)

**Question 3 [10 marks]**

a. Given $f(x)=(x+2)^2(x-1)$.

i. Identify the critical numbers and classify these critical points as local maximum, local minimum or neither. (3 marks)

ii. Determine the intervals on which the function is concave up and concave down and the $x$-coordinate of the inflection point(s). (3 marks)

b. A boat is being pulled into a dock by attached to it and passing through a pulley on the dock, positioned 6 meters higher than the boat. If the rope is being pulled in at a rate of 3 meters/sec, how fast is the boat approaching the dock when it is 8 meters from the docks? (4 marks)

**Continued...**

**Question 4 [10 marks]**

a. Given the curves $x = 2y^2$ and $x = 4 + y^2$.

i. Find the point of intersection of these curves. (2 marks)

ii. Find the area of the region bounded by these curves. (3 marks)

b. Sketch the region bounded by $y = 2 - x^2$ and $x + y = 2$. Find the volume of the solid generated by revolving the region about the $x$-axis. (5 marks)

**Question 5 [10 marks]**

a. Find the unique solution for the first order differential equation $\frac{dy}{dx} + \frac{4y}{x} = 6x - 5$ where $y(1) = 1$. (6 marks)

b. Find the general solution for the second order differential equation $y'' + 11y' + 24y = 0$. (4 marks)

**End of Paper**

**APPENDIX**

## A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$ ; $n$ is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$ ; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ ; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$ ; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$ ; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$; $x > 0$

## B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

**Integration by-parts:** $\int u\,dv = uv - \int v\,du$

**Volume (disk)** $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

**Volume (washer)** $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$